RADIATIVE HEAT TRANSFER IN A FLOW OF RHEOLOGICALLY COMPLEX FLUID

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The problem of complex radiative and convective heat transfer in steady-state generalized Couette flow of a nonlinear viscoplastic fluid is examined.

Problems of radiative and convective heat transfer require considerable effort for their solution. The complications due to the radiative component are partially due to its dependence on the geometry of the system. In view of this, it is convenient to consider a plane generalized Couette flow of fluid. The effect of geometry is easily taken into account in this case. We will carry out an analysis of complex heat transfer for a rheologically complex heat-conducting medium which attenuates (due to absorption and scattering) and emits a flux of radiant energy.

We consider a laminar flow of nonlinear viscoplastic fluid between two infinite parallel plates with a constant pressure gradient grad $p = \mathbf{A}$ in the channel. The upper isothermal plate with temperature $T^{(2)}$ moves in its own plane with constant velocity $\mathbf{V}$. The vectors $\mathbf{A}$ and $\mathbf{V}$ can have the same or opposite directions. The lower, also isothermal, plate with temperature $T^{(1)}$ is fixed. The distance between the plates is h.

We assume that the properties of the fluid are independent of temperature; we neglect heat conduction along the channel axis. We assume that the fluid is a gray medium. The scattering in it is isotropic and coherent; there is no interference nor polarization.

The y axis is normal to the plates, and the x axis coincides with the lower plate. The mathematical problem is then represented by the following system of equations: the equation of motion

$$\frac{d\tau}{dy} = \frac{dp}{dx} = A, \tag{1}$$

the rheological equation of state [1]

$$|\tau|^{\frac{1}{n}} = \tau_0^{\frac{1}{n}} + (\mu_p |\dot{\gamma}|)^{\frac{1}{m}}, \tag{2}$$

the energy transport equation

TABLE 1. Temperature Distribution in Case of a Predominant Contribution of Radiant Energy. R = 100, $\varkappa$ = 10, θ^* = 2

α	β_0	$\theta(\xi)$								
		ξ								
		0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9
0,1	0,2	1,2568	1,4137	1,5309	1,6178	1,7065	1,7780	1,8413	1,8979	1,7513
0,3	0,2	1,2569	1,4137	1,5310	1,6262	1,7072	1,7781	1,8414	1,8987	1,9513
0,5	0,2	1,2569	1,4137	1,5312	1,6264	1,7072	1,7781	1,8418	1,8988	1,9514
0,3	0,5	1,2569	1,4137	1,5310	1,6262	1,7072	1,7781	1,8414	1,8988	1,9513
0,3	0,0	1,2569	1,4136	1,5307	1,6262	1,7070	1,7780	1,8413	1,8988	1,9514
0,3	1,0	1,2569	1,4137	1,5314	1,6263	1,7074	1,7783	1,8414	1,8991	1,9521
Pure radiation		1,2568	1,4136	1,5309	1,6262	1,7072	1,7786	1,8412	1,8986	1,9512

Note: For values of ξ equal to 0 and 1, $\theta(\xi)$ is equal to 1 and 2, respectively, for pure radiation it is 1 and 2.

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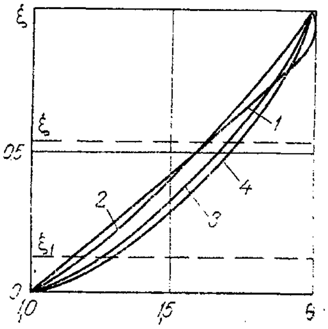

Fig. 1

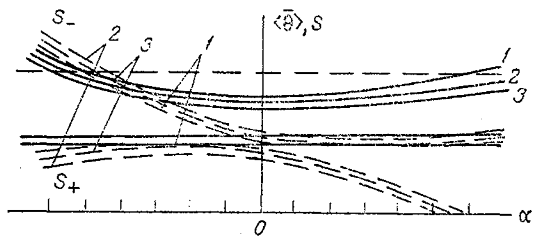

Fig. 2

Fig. 1. Effect of relative activity of radiant flux on temperature profile ($\varkappa = 10$, $\beta_0 = 0.2$, $\alpha = 0.1$, $\theta^* = 2$): 1) R = 0; 2) 0.01; 3) 1; 4) 100.

Fig. 2. Effect of radiant flux on heat-transfer rate ($\theta^* = 2$, $\varkappa = 10$): 1) Pure dissipation; 2) R = 0.01; 3) pure radiation (R = 0.01).

$$0 = \lambda \frac{d^2T}{dy^2} + \tau \frac{dV}{dy} - \frac{d}{dy} q_R. \tag{3}$$

Here τ is the shear stress; $\dot{\gamma} = dV/dy$, shear rate; τ_0, yield stress; μ_p, an analog of plastic viscosity; m, n, rheological nonlinearity parameters (both are real positive numbers); V, T, flow velocity and fluid temperature, respectively; λ, molecular thermal conductivity.

Real viscoplastic media are optically thick. Hence, to represent the radiant energy flux we use the Rosseland approximation [2]

$$q_R = -\frac{16 l^2 \sigma T^3}{3 b_R} \frac{dT}{dy}, \tag{4}$$

where l is the refractive index; σ, Stefan—Boltzmann constant; b_R, Rosseland-average attenuation coefficient, which includes the absorption and scattering coefficients.

The heat-transfer problem for a simple Couette flow of Newtonian viscous fluid has been treated in a similar way [3]. Problem (1)-(4) is closed by the boundary conditions

$$V(0) = 0,\ V(h) = U;\ T(0) = T^{(1)},\ T(h) = T^{(2)}. \tag{5}$$

Since the properties of the fluid are independent of temperature, we can solve the heat transfer and rheodynamic problems separately. Hence, to investigate the heat transfer we make use of the known results of investigation of the rheodynamic problem [1].

There are five possible, qualitatively different, flow regimes: R1 — with a quasi-solid zone (core) in the flow; R2, R3 — with the core adjacent to the lower or upper plate, respectively; R4, R5 — with the core going beyond the lower or upper plates. It has been established [1] that the flow regime is completely determined by the parameter pair (α, β_0): $\alpha = \mu_p U/(Ah)^{\frac{m}{n}} h$ is a dynamic parameter characterizing the relation between the entrainment of the fluid behind the movable plate and the pressure component of the flow; $\beta_0 = \tau_0/Ah$ is a static parameter characterizing the relation between the ultimate shear stress required to exceed the yield stress, and the actual pressure gradient in the channel.

We introduce dimensionless variables: the coordinate $\xi = y/h$, the velocity $W = V/U$, and temperature $\theta = T/T^{(1)}$. Assuming further the constancy of the fluid properties on transition from the zone of free flow to the quasi-solid core, and taking into account the signs of τ and $\dot{\gamma}$ in each flow zone, we derive from (1)-(5), after some algebra, the algebraic equations [4]

$$\theta + \frac{1}{3} R\theta^4 + \Phi_{mni}(\xi;\ \alpha,\ \beta_0,\ \varkappa) = 0. \tag{6}$$

Here $\varkappa = Ah^3(Ah)^{\frac{m}{n}}/\lambda\mu_p T^{(1)}$ is a dissipative parameter; $R = 4\sigma l (T^{(1)})^3/\lambda b_R$ is the radiation—conduction parameter, which characterizes the relation between radiation and heat conduction.

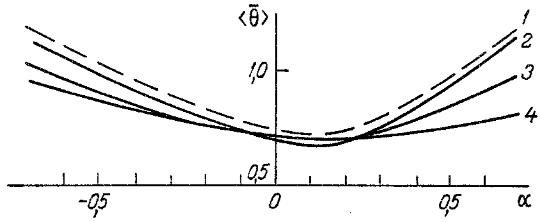

Fig. 3. Variation of mean flow temperature ($\varkappa = 10$, $\beta_0 = 0.2$, $\theta^* = 0.5$): 1) R = 0; 2) 0.01; 3) 1; 4) 100.

The form of functions Φ_{mni} depends on the flow regime. For regime R1

$$\Phi_{mn1}=\begin{cases} \varkappa\sum_{k=0}^{\infty}(-1)^k F_{mnk}(\xi_0-\xi)^{\varphi_k}+\xi(G_1-G_2)+\xi\Big[\varkappa\sum_{k=0}^{\infty}(-1)^k F_{mnk} \\ \quad\times\left\{2\varphi_k(1-\xi_0)\beta_0^{\varphi_k-1}+\xi_0^{\varphi_k}-(1-\xi_0)^{\varphi_k}\right\}\Big]-G_1- \\ \quad-\varkappa\sum_{k=0}^{\infty}(-1)^k F_{mnk}\xi_0^{\varphi_k}, \quad 0\leqslant\xi\leqslant\xi_1, \\ \xi\Big[G_1-G_2+\varkappa\sum_{k=0}^{\infty}(-1)^k F_{mnk}\left\{(1-2\xi_0)\varphi_k\beta_0^{\varphi_k-1}+\xi_0^{\varphi_k}-\right. \\ \quad\left.-(1-\xi_0)^{\varphi_k}\right\}\Big]-G_1-\varkappa\sum_{k=0}^{\infty}(-1)^k F_{mnk}\left\{-\beta_0^{\varphi_k}-\right. \\ \quad\left.-(\xi_0-\beta_0)\varphi_k\beta_0^{\varphi_k-1}+\xi_0^{\varphi_k}\right\}, \quad \xi_1\leqslant\xi\leqslant\xi_2, \\ \varkappa\sum_{k=0}^{\infty}(-1)^k F_{mnk}(\xi-\xi_0)^{\varphi_k}+\xi(G_1-G_2)+\xi\varkappa\sum_{k=0}^{\infty}(-1)^k F_{mnk}\times \\ \quad\times\{-2\varphi_k\xi_0\beta_0^{\varphi_k-1}+\xi_0^{\varphi_k}-(1-\xi_0)^{\varphi_k}\}-G_1- \\ \quad-\varkappa\sum_{k=0}^{\infty}(-1)^k F_{mnk}(-2\varphi_k\xi_0\beta_0^{\varphi_k-1}+\xi_0^{\varphi_k}), \quad \xi_2\leqslant\xi\leqslant 1. \end{cases} \tag{7}$$

For regime R2

$$\Phi_{mn2}=\begin{cases} \xi\Big[G_1-G_2+\varkappa\sum_{k=0}^{\infty}(-1)^k F_{mnk}\{\beta_0^{\varphi_k}+(1-\xi_0-\beta_0)\varphi_k\beta_0^{\varphi_k-1}- \\ \quad-(1-\xi_0)^{\varphi_k}\}\Big]-G_1, \quad 0\leqslant\xi\leqslant\xi_2, \\ \varkappa\sum_{k=0}^{\infty}(-1)^k F_{mnk}(\xi-\xi_0)^{\varphi_k}+\xi\Big[G_1-G_2+\varkappa\sum_{k=0}^{\infty}(-1)^k F_{mnk}\times \\ \quad\times\{\beta_0^{\varphi_k}-(\xi_0+\beta_0)\varphi_k\beta_0^{\varphi_k-1}-(1-\xi_0)^{\varphi_k}\}\Big]-G_1- \\ \quad-\varkappa\sum_{k=0}^{\infty}(-1)^k F_{mnk}\{\beta_0^{\varphi_k}-\varphi_k(\xi_0+\beta_0)\beta_0^{\varphi_k-1}\}, \quad \xi_2\leqslant\xi\leqslant 1. \end{cases} \tag{8}$$

For regime R3

$$\Phi_{mn3}=\begin{cases} \varkappa\sum_{k=0}^{\infty}(-1)^k F_{mnk}(\xi_0-\xi)^{\varphi_k}+\xi\Big[G_1-G_2+\varkappa\sum_{k=0}^{\infty}(-1)^k F_{mnk}\times \\ \quad\times\{-\beta_0^{\varphi_k}+\varphi_k(1-\xi_0+\beta_0)\beta_0^{\varphi_k-1}+\xi_0^{\varphi_k}\}\Big]- \\ \quad-G_1-\varkappa\sum_{k=0}^{\infty}(-1)^k F_{mnk}\xi_0^{\varphi_k}, \quad 0\leqslant\xi\leqslant\xi_1, \end{cases}$$

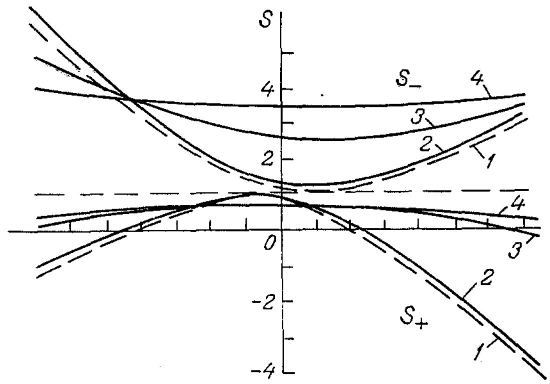

Fig. 4. Change in heat-transfer rate with increase in relative contribution of radiative heat transfer: 1) R = 0; 2) 0.01; 3) 1; 4) 100.

$$\left|\begin{array}{l}\xi\left[G_1-G_2+\varkappa\sum_{k=0}^{\infty}(-1)^k F_{mnk}\{\xi_0^{\varphi_k}-\beta_0^{\varphi_k}+(\beta_0-\xi_0)\varphi_k\beta_0^{\varphi_k-1}\}\right]-\\ -G_1-\varkappa\sum_{k=0}^{\infty}(-1)^k F_{mnk}\{\xi_0^{\varphi_k}+(\beta_0-\xi_0)\varphi_k\beta_0^{\varphi_k-1}-\beta_0^{\varphi_k}\},\ \xi_1\leqslant\xi\leqslant 1.\end{array}\right. \quad (9)$$

For regime R4

$$\Phi_{mn4}=\varkappa\sum_{k=0}^{\infty}(-1)^k F_{mnk}(\xi-\xi_0)^{\varphi_k}+\xi\left[G_1-G_2+\varkappa\sum_{k=0}^{\infty}(-1)^k F_{mnk}\right.$$

$$\left.\times\{(-\xi_0)^{\varphi_k}-(1-\xi_0)^{\varphi_k}\}\right]-G_1-\varkappa\sum_{k=0}^{\infty}(-1)^k F_{mnk}(-\xi_0)^{\varphi_k},\ \ 0\leqslant\xi\leqslant 1. \quad (10)$$

For regime R5

$$\Phi_{mn5}=\varkappa\sum_{k=0}^{\infty}(-1)^k F_{mnk}(\xi_0-\xi)^{\varphi_k}+\xi\left[G_1-G_2+\varkappa\sum_{k=0}^{\infty}(-1)^k F_{mnk}\right.$$

$$\left.\times\{\xi_0^{\varphi_k}-(\xi_0-1)^{\varphi_k}\}\right]-G_1-\varkappa\sum_{k=0}^{\infty}(-1)^k F_{mnk}\xi_0^{\varphi_k},\ \ 0\leqslant\xi\leqslant 1. \quad (11)$$

Here ξ_1, ξ_2 are the dimensionless ordinates of the quasisolid core; ξ_0 is the ordinate of the plane in which the shear stress is zero:

$$\xi_0=\xi_0(\alpha,\ \beta_0),\ \xi_1=\xi_0-\beta_0,\ \xi_2=\xi_0+\beta_0,\ G_1=1+\frac{R}{3},$$

$$G_2=\theta^*+\frac{R}{3}(\theta^*)^4,\quad F_{mnk}=\frac{c_m^k n^2\beta_0^{\frac{k}{n}}}{(m+2n-k)(m+3n-k)}, \quad (12)$$

$$c_m^k=\frac{m(m-1)\ldots(m-k+1)}{m!},\quad \varphi_k=\frac{m+3n-k}{n},\quad \theta^*=\frac{T^{(2)}}{T^{(1)}}.$$

Equations (6) are easily solved numerically by the Newton method:

$$\theta_{j+1}=\theta_j+\frac{\frac{R}{3}\theta_j^4+\theta_j+\Phi_{mnj}}{\frac{4}{3}R\theta_j^3+1},\ \ j=0,\ 1,\ \ldots \quad (13)$$

and, since $\theta(\xi)>0$, $0\leq\xi\leq 1$, the denominator of the fraction in (13) does not become zero (j is the iteration number).

To calculate the temperature gradients on the walls we have the formulas

$$\left.\frac{d\theta}{d\xi}\right|_{\xi=0}=\frac{-\frac{d}{d\xi}\Phi_{mni}(0)}{1+\frac{4}{3}R},\quad \left.\frac{d\theta}{d\xi}\right|_{\xi=1}=\frac{-\frac{d}{d\xi}\Phi_{mni}(1)}{1+\frac{4}{3}R(\theta^*)^3}. \tag{14}$$

We consider separately three cases: $R \ll 1$, $R \sim 1$, $R \gg 1$. In the first case molecular heat conduction predominates, in the second case energy transfer by conduction and radiation is of the same order of magnitude, and in the third case radiation predominates. When $R \to 0$ the temperature profiles are the same as for purely dissipative heat transfer [5]. When $R \to \infty$ the temperature field is determined by the second term on the left-hand side of (6). The effect of dissipative heat release is negligible (irrespective of the values of m and n) for $R:R/\varkappa \geq 10$ (see Table 1).

The radiative component leads to a qualitative change in the temperature distributions. The temperature profiles for purely dissipative heat transfer in a flow of nonlinear visco-plastic fluid at high shear rates are characterized by convexity, where the fluid temperature exceeds the temperature of the hotter plate [5]. For $R > 0$ the temperature profiles are smoother and do not exceed θ^*, i.e., radiative transfer promotes a more uniform temperature distribution in the channel (Fig. 1).

Instead of the traditional Nusselt number Nu, it is more convenient to use the "heat-transfer rate" [5]:

$$S=\frac{h}{T^{(2)}-T^{(1)}}\left(\frac{dT}{dy}\right)_w. \tag{15}$$

and the heat-transfer characteristic. Conversion from S to Nu is simple:

$$\mathrm{Nu}=\frac{T^{(2)}-T^{(1)}}{\langle T\rangle-T_w}S, \tag{16}$$

where $\langle T\rangle = \int_0^h T(y)V(y)\,dy/\int_0^h V(y)\,dy$ is the mean flow temperature; the subscript w indicates a wall value.

The heat-transfer dependence on the relative contribution of the radiant energy flux is shown in Figs. 2-4; S_- is the heat-transfer rate on the lower plate, and S_+ is that on the upper plate.

Figures 2 and 3 shows an interesting feature: When a radiant energy flux appears ($R > 0$) in a fluid with constant properties and its contribution to the total heat-transfer process increases (with increase in R) the mean flow temperature $\langle\theta\rangle$ (continuous line in Fig. 2) decreases and $\langle\theta\rangle$ becomes less dependent on α, β_0 (compare with [5]). As already mentioned, the radiant flux changes the temperature distribution over the channel cross section, lowering the temperature in regions of high shear rate, and increasing it in regions of low rate. Since the appearance of the additional radiant energy flux (and its increase) has no effect on the rheodynamics, the mean velocity

$$\langle W\rangle=\int_0^1 W(\xi)\,d\xi=\mathrm{const}(R) \tag{17}$$

for a given set of values of the parameters (α, β_0). The determination of $\langle\theta\rangle$ involves another quantity

$$\vartheta=\int_0^1 \theta(\xi)W(\xi)\,d\xi. \tag{18}$$

Since the temperature for the region of high velocities decreases with increase in R, and increases at low velocities, then, generally speaking, ϑ will be a decreasing function of R. The same applies for $\langle\theta\rangle=\vartheta/\langle W\rangle$. The reduction of $\langle\theta\rangle$ with increase in R is more appreciable for $\alpha > 0$ than for $\alpha < 0$.

The redistribution ("smoothing") of the temperature greatly affects the heat transfer rate. The curves of $S(\alpha, \beta_0)$, in comparison with the case $R = 0$ become smoother and less steep (Fig. 4). In the region of low α, where the relative magnitude of the heat flux due to dissipation of mechanical energy is small, $|S|$ increases with increase in R. In the region of larger α, where dissipative heat release is significant, the "smoothing" of the temperature leads to reduction of $|S|$ with increase in R. When $R \to \infty$ the shape of the $S(\alpha)$ curves tends to a straight line parallel to the axis 0α. This conclusion can also be directly derived from an analysis of Eqs. (6) and (14).

The solution of the problem of complex heat transfer for a simple Couette flow of Newtonian viscous fluid [3] is obtained from the solution given in this paper as the special case where $m = n = 1$, $\beta_0 = 0$, $\alpha \to \infty$, and the dissipative function $\tau dV/dy$ is averaged over the cross section.

NOTATION

p, pressure; T, temperature; V, flow velocity; τ, tangential shear stress; $\dot{\gamma}$, shear rate; y, coordinate normal to plates; α, β_0, $\varkappa$, R, dimensionless parameters; θ^*, ratio of temperature of upper plate to that of lower; λ, thermal conductivity; q_R, radiation flux density.

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